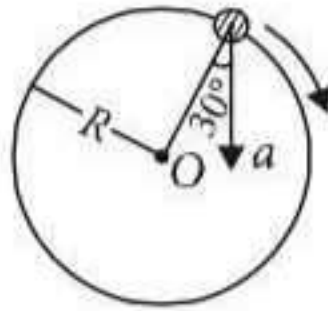


Chapter 3. Motion in a Plane

1. The x and y coordinates of the particle at any time are $x = 5t - 2t^2$ and $y = 10t$ respectively, where x and y are in metres and t in seconds. The acceleration of the particle at $t = 2$ s is
- (a) 5 m s^{-2} (b) -4 m s^{-2}
 (c) -8 m s^{-2} (d) 0 (NEET 2017)

2. In the given figure, $a = 15 \text{ m s}^{-2}$ represents the total acceleration of a particle moving in the clockwise direction in a circle of radius $R = 2.5 \text{ m}$ at a given instant of time. The speed of the particle is



- (a) 4.5 m s^{-1} (b) 5.0 m s^{-1}
 (c) 5.7 m s^{-1} (d) 6.2 m s^{-1}
 (NEET-II 2016)

3. If the magnitude of sum of two vectors is equal to the magnitude of difference of the two vectors, the angle between these vectors is
- (a) 45° (b) 180°
 (c) 0° (d) 90° (NEET-I 2016)

4. A particle moves so that its position vector is given by $\vec{r} = \cos \omega t \hat{x} + \sin \omega t \hat{y}$, where ω is a constant.

Which of the following is true?

- (a) Velocity is perpendicular to \vec{r} and acceleration is directed towards the origin.
 (b) Velocity is perpendicular to \vec{r} and acceleration is directed away from the origin.
 (c) Velocity and acceleration both are perpendicular to \vec{r}
 (d) Velocity and acceleration both are parallel to \vec{r} (NEET-I 2016)

5. If vectors $\vec{A} = \cos \omega t \hat{i} + \sin \omega t \hat{j}$ and

$$\vec{B} = \cos \frac{\omega t}{2} \hat{i} + \sin \frac{\omega t}{2} \hat{j}$$

are functions of time,

then the value of t at which they are orthogonal to each other is

- (a) $t = \frac{\pi}{\omega}$ (b) $t = 0$
 (c) $t = \frac{\pi}{4\omega}$ (d) $t = \frac{\pi}{2\omega}$ (2015)

6. The position vector of a particle \vec{R} as a function of time is given by $\vec{R} = 4 \sin(2\pi t) \hat{i} + 4 \cos(2\pi t) \hat{j}$. Where R is in meters, t is in seconds and \hat{i} and \hat{j} denote unit vectors along x - and y -directions, respectively. Which one of the following statements is wrong for the motion of particle?

- (a) Magnitude of the velocity of particle is 8 meter/second.
 (b) Path of the particle is a circle of radius 4 meter.
 (c) Acceleration vector is along $-\vec{R}$.
 (d) Magnitude of acceleration vector is $\frac{v^2}{R}$, where v is the velocity of particle. (2015)

7. A ship A is moving Westwards with a speed of 10 km h^{-1} and a ship B 100 km South of A , is moving Northwards with a speed of 10 km h^{-1} . The time after which the distance between them becomes shortest, is

- (a) $5\sqrt{2} \text{ h}$ (b) $10\sqrt{2} \text{ h}$
 (c) 0 h (d) 5 h
 (2015 Cancelled)

8. A projectile is fired from the surface of the earth with a velocity of 5 m s^{-1} and angle θ with the horizontal. Another projectile fired from another planet with a velocity of 3 m s^{-1} at the same angle follows a trajectory which is identical with the trajectory of the projectile fired from the earth. The value of the acceleration due to gravity on the planet is (in m s^{-2}) is

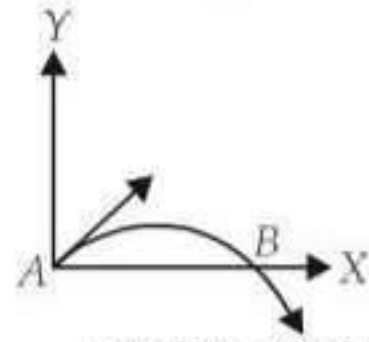
- (Given $g = 9.8 \text{ m s}^{-2}$)
 (a) 3.5 (b) 5.9 (c) 16.3 (d) 110.8
 (2014)

9. A particle is moving such that its position coordinates (x, y) are $(2 \text{ m}, 3 \text{ m})$ at time $t = 0$, $(6 \text{ m}, 7 \text{ m})$ at time $t = 2 \text{ s}$ and $(13 \text{ m}, 14 \text{ m})$ at time $t = 5 \text{ s}$. Average velocity vector (\vec{v}_{av}) from $t = 0$ to $t = 5 \text{ s}$ is

- (a) $\frac{1}{5}(13\hat{i} + 14\hat{j})$ (b) $\frac{7}{3}(\hat{i} + \hat{j})$
 (c) $2(\hat{i} + \hat{j})$ (d) $\frac{11}{5}(\hat{i} + \hat{j})$ (2014)

10. The velocity of a projectile at the initial point A is $(2\hat{i} + 3\hat{j}) \text{ m/s}$. Its velocity (in m/s) at point B is

- (a) $2\hat{i} - 3\hat{j}$
 (b) $2\hat{i} + 3\hat{j}$
 (c) $-2\hat{i} - 3\hat{j}$
 (d) $-2\hat{i} + 3\hat{j}$



(NEET 2013)

11. Vectors \vec{A} , \vec{B} and \vec{C} are such that $\vec{A} \cdot \vec{B} = 0$ and $\vec{A} \cdot \vec{C} = 0$. Then the vector parallel to \vec{A} is

- (a) $\vec{A} \times \vec{B}$ (b) $\vec{B} + \vec{C}$
 (c) $\vec{B} \times \vec{C}$ (d) \vec{B} and \vec{C}

(Karnataka NEET 2013)

12. The horizontal range and the maximum height of a projectile are equal. The angle of projection of the projectile is

- (a) $\theta = \tan^{-1}\left(\frac{1}{4}\right)$ (b) $\theta = \tan^{-1}(4)$
 (c) $\theta = \tan^{-1}(2)$ (d) $\theta = 45^\circ$ (2012)

13. A particle has initial velocity $(2\vec{i} + 3\vec{j})$ and acceleration $(0.3\vec{i} + 0.2\vec{j})$. The magnitude of velocity after 10 seconds will be

- (a) $9\sqrt{2}$ units (b) $5\sqrt{2}$ units
 (c) 5 units (d) 9 units (2012)

14. A particle moves in a circle of radius 5 cm with constant speed and time period $0.2\pi \text{ s}$. The acceleration of the particle is

- (a) 15 m/s^2 (b) 25 m/s^2
 (c) 36 m/s^2 (d) 5 m/s^2 (2011)

15. A missile is fired for maximum range with an initial velocity of 20 m/s . If $g = 10 \text{ m/s}^2$, the range of the missile is

- (a) 40 m (b) 50 m
 (c) 60 m (d) 20 m (2011)

16. A body is moving with velocity 30 m/s towards east. After 10 seconds its velocity becomes 40 m/s towards north. The average acceleration of the body is

- (a) 1 m/s^2 (b) 7 m/s^2
 (c) $\sqrt{7} \text{ m/s}^2$ (d) 5 m/s^2 (2011)

17. A projectile is fired at an angle of 45° with the horizontal. Elevation angle of the projectile at its highest point as seen from the point of projection, is

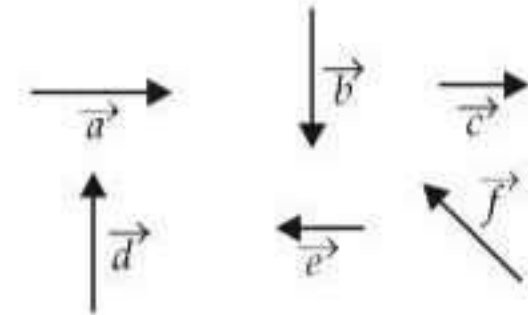
- (a) 45° (b) 60°
 (c) $\tan^{-1}\left(\frac{1}{2}\right)$ (d) $\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(Mains 2011)

18. A particle has initial velocity $(3\hat{i} + 4\hat{j})$ and has acceleration $(0.4\hat{i} + 0.3\hat{j})$. Its speed after 10 s is

- (a) 7 units (b) $7\sqrt{2}$ units
 (c) 8.5 units (d) 10 units (2010)

19. Six vectors \vec{a} through \vec{f} have the magnitudes and directions indicated in the figure. Which of the following statements is true?



- (a) $\vec{b} + \vec{c} = \vec{f}$ (b) $\vec{d} + \vec{c} = \vec{f}$
 (c) $\vec{d} + \vec{e} = \vec{f}$ (d) $\vec{b} + \vec{e} = \vec{f}$ (2010)

20. The speed of a projectile at its maximum height is half of its initial speed. The angle of projection is

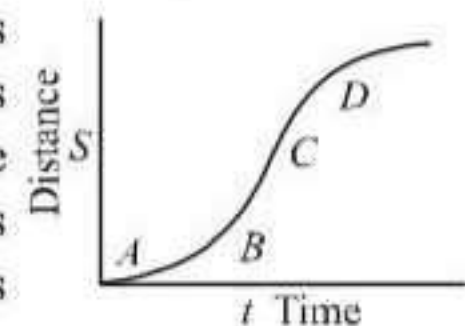
- (a) 60° (b) 15° (c) 30° (d) 45°

(Mains 2010)

21. A particle moves in x - y plane according to rule $x = a \sin \omega t$ and $y = a \cos \omega t$. The particle follows

- (a) an elliptical path
 (b) a circular path
 (c) a parabolic path
 (d) a straight line path inclined equally to x and y -axes (Mains 2010)

22. A particle shows distance - time curve as given in this figure. The maximum instantaneous velocity of the particle is around the point



- (a) D (b) A
 (c) B (d) C (2008)

23. A particle of mass m is projected with velocity v making an angle of 45° with the horizontal.

When the particle lands on the level ground the magnitude of the change in its momentum will be

- (a) $mv\sqrt{2}$ (b) zero
(c) $2mv$ (d) $mv/\sqrt{2}$ (2008)

24. \vec{A} and \vec{B} are two vectors and θ is the angle between them, if $|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A} \cdot \vec{B})$, the value of θ is

- (a) 45° (b) 30°
(c) 90° (d) 60° (2007)

25. A particle starting from the origin (0, 0) moves in a straight line in the (x, y) plane. Its coordinates at a later time are $(\sqrt{3}, 3)$. The path of the particle makes with the x-axis an angle of

- (a) 45° (b) 60°
(c) 0° (d) 30° (2007)

26. For angles of projection of a projectile at angle $(45^\circ - \theta)$ and $(45^\circ + \theta)$, the horizontal range described by the projectile are in the ratio of

- (a) 2 : 1 (b) 1 : 1
(c) 2 : 3 (d) 1 : 2 (2006)

27. The vectors \vec{A} and \vec{B} are such that $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$. The angle between the two vectors is

- (a) 45° (b) 90° (c) 60° (d) 75°
(2006, 1996, 1991)

28. Two boys are standing at the ends A and B of a ground where $AB = a$. The boy at B starts running in a direction perpendicular to AB with velocity v_1 . The boy at A starts running simultaneously with velocity v and catches the other in a time t , where t is

- (a) $\frac{a}{\sqrt{v^2 + v_1^2}}$ (b) $\frac{a}{v + v_1}$
(c) $\frac{a}{v - v_1}$ (d) $\frac{a^2}{\sqrt{v^2 - v_1^2}}$
(2005)

29. A stone tied to the end of a string of 1 m long is whirled in a horizontal circle with a constant speed. If the stone makes 22 revolutions in 44 seconds, what is the magnitude and direction of acceleration of the stone?

- (a) $\pi^2 \text{ m s}^{-2}$ and direction along the radius towards the centre
(b) $\pi^2 \text{ m s}^{-2}$ and direction along the radius away

from the centre

- (c) $\pi^2 \text{ m s}^{-2}$ and direction along the tangent to the circle
(d) $\pi^2/4 \text{ ms}^{-2}$ and direction along the radius towards the centre. (2005)

30. If the angle between the vectors \vec{A} and \vec{B} is θ , the value of the product $(\vec{B} \times \vec{A}) \cdot \vec{A}$ is equal to

- (a) $BA^2 \sin \theta$ (b) $BA^2 \cos \theta$
(c) $BA^2 \sin \theta \cos \theta$ (d) zero.
(2005, 1989)

31. If a vector $2\hat{i} + 3\hat{j} + 8\hat{k}$ is perpendicular to the vector $4\hat{j} - 4\hat{i} + \alpha\hat{k}$, then the value of α is

- (a) 1/2 (b) -1/2
(c) 1 (d) -1. (2005)

32. If $|\vec{A} \times \vec{B}| = \sqrt{3}\vec{A} \cdot \vec{B}$ then the value of $|\vec{A} + \vec{B}|$ is

- (a) $(A^2 + B^2 + AB)^{1/2}$
(b) $(A^2 + B^2 + \frac{AB}{\sqrt{3}})^{1/2}$
(c) $A + B$
(d) $(A^2 + B^2 + \sqrt{3}AB)^{1/2}$. (2004)

33. The vector sum of two forces is perpendicular to their vector differences. In that case, the forces

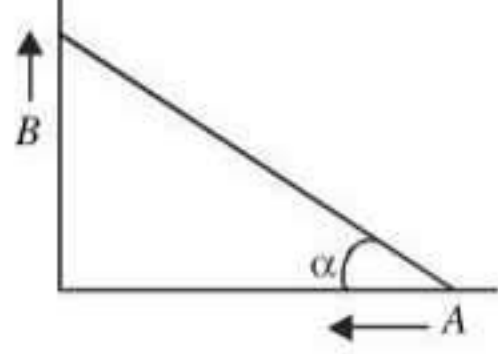
- (a) are equal to each other
(b) are equal to each other in magnitude
(c) are not equal to each other in magnitude
(d) cannot be predicted. (2003)

34. A particle moves along a circle of radius $(\frac{20}{\pi}) \text{ m}$ with constant tangential acceleration. If the velocity of the particle is 80 m/s at the end of the second revolution after motion has begun, the tangential acceleration is

- (a) 40 m/s^2 (b) $640\pi \text{ m/s}^2$
(c) $160\pi \text{ m/s}^2$ (d) $40\pi \text{ m/s}^2$. (2003)

35. A particle A is dropped from a height and another particle B is projected in horizontal direction with speed of 5 m/sec from the same height then correct statement is

- (a) particle A will reach at ground first with respect to particle B
(b) particle B will reach at ground first with respect to particle A
(c) both particles will reach at ground simultaneously
(d) both particles will reach at ground with

- same speed. (2002)
36. An object of mass 3 kg is at rest. Now a force of $\vec{F} = 6t^2\hat{i} + 4t\hat{j}$ is applied on the object then velocity of object at $t = 3$ sec. is
 (a) $18\hat{i} + 3\hat{j}$ (b) $18\hat{i} + 6\hat{j}$
 (c) $3\hat{i} + 18\hat{j}$ (d) $18\hat{i} + 4\hat{j}$. (2002)
37. If $|\vec{A} + \vec{B}| = |\vec{A}| + |\vec{B}|$ then angle between A and B will be
 (a) 90° (b) 120°
 (c) 0° (d) 60° . (2001)
38. Two particles having mass M and m are moving in a circular path having radius R and r . If their time period are same then the ratio of angular velocity will be
 (a) $\frac{r}{R}$ (b) $\frac{R}{r}$
 (c) 1 (d) $\sqrt{\frac{R}{r}}$. (2001)
39. The width of river is 1 km. The velocity of boat is 5 km/hr. The boat covered the width of river in shortest time 15 min. Then the velocity of river stream is
 (a) 3 km/hr (b) 4 km/hr
 (c) $\sqrt{29}$ km/hr (d) $\sqrt{41}$ km/hr. (2000, 1998)
40. Two projectiles of same mass and with same velocity are thrown at an angle 50° and 30° with the horizontal, then which will remain same
 (a) time of flight
 (b) range of projectile
 (c) maximum height acquired
 (d) all of them. (2000)
41. A man is slipping on a frictionless inclined plane and a bag falls down from the same height. Then the velocity of both is related as
 (a) $v_B > v_m$
 (b) $v_B < v_m$
 (c) $v_B = v_m$
 (d) v_B and v_m can't be related. (2000)
42. A 500 kg car takes a round turn of radius 50 m with a velocity of 36 km/hr. The centripetal force is
 (a) 1000 N (b) 750 N
 (c) 250 N (d) 1200 N (1999)
43. A person aiming to reach exactly opposite point on the bank of a stream is swimming with a speed of 0.5 m/s at an angle of 120° with the direction of flow of water. The speed of water in the stream, is
 (a) 0.25 m/s (b) 0.5 m/s
 (c) 1.0 m/s (d) 0.433 m/s (1999)
44. Two racing cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 respectively. Their speeds are such that each makes a complete circle in the same time t . The ratio of the angular speeds of the first to the second car is
 (a) $r_1 : r_2$ (b) $m_1 : m_2$
 (c) 1 : 1 (d) $m_1 m_2 : r_1 r_2$. (1999)
45. If a unit vector is represented by $0.5\hat{i} - 0.8\hat{j} + c\hat{k}$ then the value of c is
 (a) $\sqrt{0.01}$ (b) $\sqrt{0.11}$
 (c) 1 (d) $\sqrt{0.39}$. (1999)
46. What is the value of linear velocity, if $\vec{r} = 3\hat{i} - 4\hat{j} + \hat{k}$ and $\vec{\omega} = 5\hat{i} - 6\hat{j} + 6\hat{k}$?
 (a) $4\hat{i} - 13\hat{j} + 6\hat{k}$ (b) $18\hat{i} + 13\hat{j} - 2\hat{k}$
 (c) $6\hat{i} + 2\hat{j} - 3\hat{k}$ (d) $6\hat{i} - 2\hat{j} + 8\hat{k}$. (1999)
47. Two particles A and B are connected by a rigid rod AB . The rod slides along perpendicular rails as shown here. The velocity of A to the left is 10 m/s. What is the velocity of B when angle $\alpha = 60^\circ$?
 (a) 10 m/s (b) 9.8 m/s
 (c) 5.8 m/s (d) 17.3 m/s. (1998)
- 
48. A ball of mass 0.25 kg attached to the end of a string of length 1.96 m is moving in a horizontal circle. The string will break if the tension is more than 25 N. What is the maximum speed with which the ball can be moved?
 (a) 5 m/s (b) 3 m/s
 (c) 14 m/s (d) 3.92 m/s. (1998)
49. Identify the vector quantity among the following
 (a) distance (b) angular momentum
 (c) heat (d) energy. (1997)
50. A body is whirled in a horizontal circle of radius 20 cm. It has an angular velocity of 10 rad/s. What is its linear velocity at any point on circular path?
 (a) 20 m/s (b) $\sqrt{2}$ m/s
 (c) 10 m/s (d) 2 m/s. (1996)
51. The position vector of a particle is $\vec{r} = (a\cos\omega t)\hat{i} + (a\sin\omega t)\hat{j}$. The velocity of

- the particle is
 (a) directed towards the origin
 (b) directed away from the origin
 (c) parallel to the position vector
 (d) perpendicular to the position vector. (1995)
52. The angular speed of a flywheel making 120 revolutions/minute is
 (a) 4π rad/s (b) $4\pi^2$ rad/s
 (c) π rad/s (d) 2π rad/s. (1995)
53. The angle between the two vectors $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$ will be
 (a) 90° (b) 180°
 (c) zero (d) 45° . (1994)
54. A boat is sent across a river with a velocity of 8 km h^{-1} . If the resultant velocity of boat is 10 km h^{-1} , then velocity of river is
 (a) 12.8 km h^{-1} (b) 6 km h^{-1}
 (c) 8 km h^{-1} (d) 10 km h^{-1} (1994, 1993)
55. If a body A of mass M is thrown with velocity v at an angle of 30° to the horizontal and another body B of the same mass is thrown with the same speed at an angle of 60° to the horizontal, the ratio of horizontal range of A to B will be
 (a) 1 : 3 (b) 1 : 1
 (c) $1 : \sqrt{3}$ (d) $\sqrt{3} : 1$. (1992, 90)
56. The resultant of $\vec{A} \times \vec{0}$ will be equal to
 (a) zero (b) A
 (c) zero vector (d) unit vector. (1992)
57. An electric fan has blades of length 30 cm measured from the axis of rotation. If the fan is rotating at 120 rpm, the acceleration of a point on the tip of the blade is
 (a) 1600 m s^{-2} (b) 47.4 m s^{-2}
 (c) 23.7 m s^{-2} (d) 50.55 m s^{-2} (1990)
58. The maximum range of a gun of horizontal terrain is 16 km. If $g = 10 \text{ ms}^{-2}$, then muzzle velocity of a shell must be
 (a) 160 m s^{-1} (b) $200\sqrt{2} \text{ m s}^{-1}$
 (c) 400 m s^{-1} (d) 800 m s^{-1} (1990)
59. A bus is moving on a straight road towards north with a uniform speed of 50 km/hour then it turns left through 90° . If the speed remains unchanged after turning, the increase in the velocity of bus in the turning process is
 (a) 70.7 km/hr along south-west direction
 (b) zero
 (c) 50 km/hr along west
 (d) 70.7 km/hr along north-west direction (1989)
60. The magnitude of vectors \vec{A} , \vec{B} and \vec{C} are 3, 4 and 5 units respectively. If $\vec{A} + \vec{B} = \vec{C}$, the angle between \vec{A} and \vec{B} is
 (a) $\pi/2$ (b) $\cos^{-1}(0.6)$
 (c) $\tan^{-1}(7/5)$ (d) $\pi/4$. (1988)
61. A train of 150 metre length is going towards north direction at a speed of 10 m/s. A parrot flies at the speed of 5 m/s towards south direction parallel to the railways track. The time taken by the parrot to cross the train is
 (a) 12 sec (b) 8 sec
 (c) 15 sec (d) 10 sec. (1988)

Answer Key

1. (b) 2. (c) 3. (d) 4. (a) 5. (a) 6. (a) 7. (d) 8. (a) 9. (d) 10. (a)
 11. (c) 12. (b) 13. (b) 14. (d) 15. (a) 16. (d) 17. (c) 18. (b) 19. (c) 20. (a)
 21. (b) 22. (d) 23. (a) 24. (d) 25. (b) 26. (b) 27. (b) 28. (d) 29. (a, b) 30. (d)
 31. (b) 32. (a) 33. (b) 34. (a) 35. (c) 36. (b) 37. (c) 38. (c) 39. (a) 40. (b)
 41. (c) 42. (a) 43. (a) 44. (c) 45. (b) 46. (b) 47. (d) 48. (c) 49. (b) 50. (d)
 51. (d) 52. (a) 53. (a) 54. (b) 55. (b) 56. (c) 57. (b) 58. (c) 59. (a) 60. (a)
 61. (d)



EXPLANATIONS

1. (b) : $x = 5t - 2t^2, y = 10t$

$$\frac{dx}{dt} = 5 - 4t, \quad \frac{dy}{dt} = 10 \quad \therefore v_x = 5 - 4t, v_y = 10$$

$$\frac{dv_x}{dt} = -4, \quad \frac{dv_y}{dt} = 0 \quad \therefore a_x = -4, a_y = 0$$

Acceleration, $\vec{a} = a_x \hat{i} + a_y \hat{j} = -4\hat{i}$

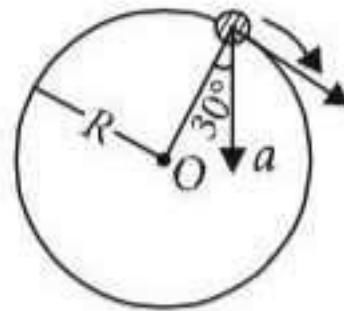
\therefore The acceleration of the particle at $t = 2$ s is -4 m s^{-2} .

2. (c) : Here, $a = 15 \text{ m s}^{-2}$

$$R = 2.5 \text{ m}$$

From figure,

$$a_c = a \cos 30^\circ = 15 \times \frac{\sqrt{3}}{2} \text{ m s}^{-2}$$



As we know, $a_c = \frac{v^2}{R} \Rightarrow v = \sqrt{a_c R}$

$$\therefore v = \sqrt{15 \times \frac{\sqrt{3}}{2} \times 2.5} = 5.69 \approx 5.7 \text{ m s}^{-1}$$

3. (d) : Let the two vectors be \vec{A} and \vec{B} .

Then, magnitude of sum of \vec{A} and \vec{B} ,

$$|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

and magnitude of difference of \vec{A} and \vec{B} ,

$$|\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - 2AB \cos \theta}$$

$$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}| \quad (\text{given})$$

or $\sqrt{A^2 + B^2 + 2AB \cos \theta} = \sqrt{A^2 + B^2 - 2AB \cos \theta}$

$$\Rightarrow 4AB \cos \theta = 0$$

$$\therefore 4AB \neq 0, \quad \therefore \cos \theta = 0 \text{ or } \theta = 90^\circ$$

4. (a) : Given, $\vec{r} = \cos \omega t \hat{x} + \sin \omega t \hat{y}$

$$\therefore \vec{v} = \frac{d\vec{r}}{dt} = -\omega \sin \omega t \hat{x} + \omega \cos \omega t \hat{y}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -\omega^2 \cos \omega t \hat{x} - \omega^2 \sin \omega t \hat{y} = -\omega^2 \vec{r}$$

Since position vector (\vec{r}) is directed away from the origin, so, acceleration ($-\omega^2 \vec{r}$) is directed towards the origin.

Also,

$$\vec{r} \cdot \vec{v} = (\cos \omega t \hat{x} + \sin \omega t \hat{y}) \cdot (-\omega \sin \omega t \hat{x} + \omega \cos \omega t \hat{y})$$

$$= -\omega \sin \omega t \cos \omega t + \omega \sin \omega t \cos \omega t = 0$$

$$\Rightarrow \vec{r} \perp \vec{v}$$

5. (a) : Two vectors \vec{A} and \vec{B} are orthogonal to each other, if their scalar product is zero i.e. $\vec{A} \cdot \vec{B} = 0$.

Here, $\vec{A} = \cos \omega t \hat{i} + \sin \omega t \hat{j}$

and $\vec{B} = \cos \frac{\omega t}{2} \hat{i} + \sin \frac{\omega t}{2} \hat{j}$

$$\therefore \vec{A} \cdot \vec{B} = (\cos \omega t \hat{i} + \sin \omega t \hat{j}) \cdot \left(\cos \frac{\omega t}{2} \hat{i} + \sin \frac{\omega t}{2} \hat{j} \right)$$

$$= \cos \omega t \cos \frac{\omega t}{2} + \sin \omega t \sin \frac{\omega t}{2}$$

$$(\because \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = 1 \text{ and } \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{i} = 0)$$

$$= \cos \left(\omega t - \frac{\omega t}{2} \right)$$

$$(\because \cos(A - B) = \cos A \cos B + \sin A \sin B)$$

But $\vec{A} \cdot \vec{B} = 0$ (as \vec{A} and \vec{B} are orthogonal to each other)

$$\therefore \cos \left(\omega t - \frac{\omega t}{2} \right) = 0$$

$$\cos \left(\omega t - \frac{\omega t}{2} \right) = \cos \frac{\pi}{2} \text{ or } \omega t - \frac{\omega t}{2} = \frac{\pi}{2}$$

$$\frac{\omega t}{2} = \frac{\pi}{2} \text{ or } t = \frac{\pi}{\omega}$$

6. (a) : Here, $\vec{R} = 4 \sin(2\pi t) \hat{i} + 4 \cos(2\pi t) \hat{j}$

The velocity of the particle is

$$\vec{v} = \frac{d\vec{R}}{dt} = \frac{d}{dt} [4 \sin(2\pi t) \hat{i} + 4 \cos(2\pi t) \hat{j}]$$

$$= 8\pi \cos(2\pi t) \hat{i} - 8\pi \sin(2\pi t) \hat{j}$$

Its magnitude is

$$|\vec{v}| = \sqrt{(8\pi \cos(2\pi t))^2 + (-8\pi \sin(2\pi t))^2}$$

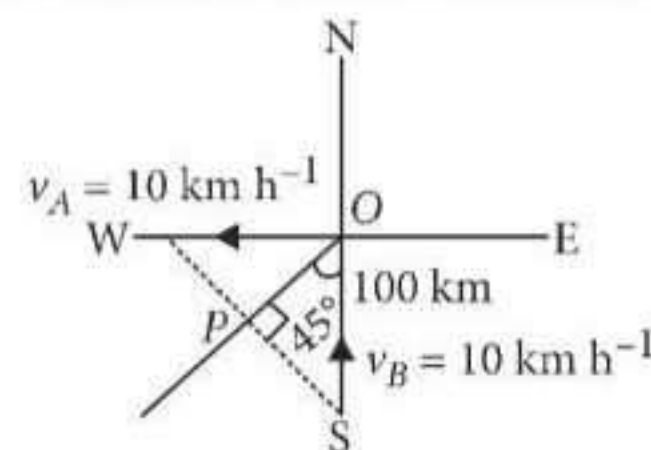
$$= \sqrt{64\pi^2 \cos^2(2\pi t) + 64\pi^2 \sin^2(2\pi t)}$$

$$= \sqrt{64\pi^2 [\cos^2(2\pi t) + \sin^2(2\pi t)]}$$

$$= \sqrt{64\pi^2} \quad (\text{as } \sin^2 \theta + \cos^2 \theta = 1)$$

$$= 8\pi \text{ m/s}$$

7. (d) : Given situation is shown in the figure.



Velocity of ship A,

$$v_A = 10 \text{ km h}^{-1} \text{ towards west}$$

Velocity of ship B,

$$v_B = 10 \text{ km h}^{-1} \text{ towards north}$$

$$OS = 100 \text{ km}$$

OP = shortest distance

Relative velocity between A and B is

$$v_{AB} = \sqrt{v_A^2 + v_B^2} = 10\sqrt{2} \text{ km h}^{-1}$$

$$\cos 45^\circ = \frac{OP}{OS}; \frac{1}{\sqrt{2}} = \frac{OP}{100}$$

$$OP = \frac{100}{\sqrt{2}} = \frac{100\sqrt{2}}{2} = 50\sqrt{2} \text{ km}$$

The time after which distance between them equals to OP is given by

$$t = \frac{OP}{v_{AB}} = \frac{50\sqrt{2}}{10\sqrt{2}} \Rightarrow t = 5 \text{ h}$$

8. (a) : The equation of trajectory is

$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

where θ is the angle of projection and u is the velocity with which projectile is projected.

For equal trajectories and for same angles of projection,

$$\frac{g}{u^2} = \text{constant}$$

As per question, $\frac{9.8}{5^2} = \frac{g'}{3^2}$

where g' is acceleration due to gravity on the planet.

$$g' = \frac{9.8 \times 9}{25} = 3.5 \text{ m s}^{-2}$$

9. (d) : At time $t = 0$, the position vector of the particle is

$$\vec{r}_1 = 2\hat{i} + 3\hat{j}$$

At time $t = 5$ s, the position vector of the particle is

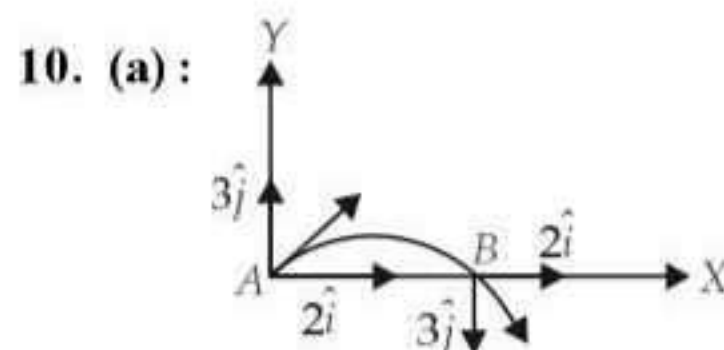
$$\vec{r}_2 = 13\hat{i} + 14\hat{j}$$

Displacement from \vec{r}_1 to \vec{r}_2 is

$$\Delta \vec{r} = \vec{r}_2 - \vec{r}_1 = (13\hat{i} + 14\hat{j}) - (2\hat{i} + 3\hat{j}) = 11\hat{i} + 11\hat{j}$$

\therefore Average velocity,

$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t} = \frac{11\hat{i} + 11\hat{j}}{5 - 0} = \frac{11}{5}(\hat{i} + \hat{j})$$



10. (a) :

At point B X component of velocity remains unchanged while Y component reverses its direction.

\therefore The velocity of the projectile at point B is $2\hat{i} - 3\hat{j}$ m/s.

11. (c) : Vector triple product of three vectors \vec{A} , \vec{B} and \vec{C} is

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A} \cdot \vec{C})\vec{B} - (\vec{A} \cdot \vec{B})\vec{C}$$

Given: $\vec{A} \cdot \vec{B} = 0$, $\vec{A} \cdot \vec{C} = 0$

$$\therefore \vec{A} \times (\vec{B} \times \vec{C}) = 0$$

Thus the vector \vec{A} is parallel to vector $\vec{B} \times \vec{C}$.

12. (b) : Horizontal range, $R = \frac{u^2 \sin 2\theta}{g}$

where u is the velocity of projection and θ is the angle of projection

Maximum height, $H = \frac{u^2 \sin^2 \theta}{2g}$

According to question $R = H$

$$\therefore \frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2 \theta}{2g}$$

$$\frac{2u^2 \sin \theta \cos \theta}{g} = \frac{u^2 \sin^2 \theta}{2g}$$

$$\tan \theta = 4 \text{ or } \theta = \tan^{-1}(4)$$

13. (b) : Here, $\vec{u} = 2\hat{i} + 3\hat{j}$, $\vec{a} = 0.3\hat{i} + 0.2\hat{j}$, $t = 10$ s

As $\vec{v} = \vec{u} + \vec{a}t$

$$\therefore \vec{v} = (2\hat{i} + 3\hat{j}) + (0.3\hat{i} + 0.2\hat{j})(10)$$

$$= 2\hat{i} + 3\hat{j} + 3\hat{i} + 2\hat{j} = 5\hat{i} + 5\hat{j}$$

$$|\vec{v}| = \sqrt{(5)^2 + (5)^2} = 5\sqrt{2} \text{ units}$$

14. (d) : Here, Radius, $R = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$

Time period, $T = 0.2\pi$ s

Centripetal acceleration

$$a_c = \omega^2 R = \left(\frac{2\pi}{T}\right)^2 R = \left(\frac{2\pi}{0.2\pi}\right)^2 (5 \times 10^{-2}) = 5 \text{ m/s}^2$$

As particle moves with constant speed, therefore its tangential acceleration is zero. So, $a_t = 0$

The acceleration of the particle is

$$a = \sqrt{a_c^2 + a_t^2} = a_c = 5 \text{ m/s}^2$$

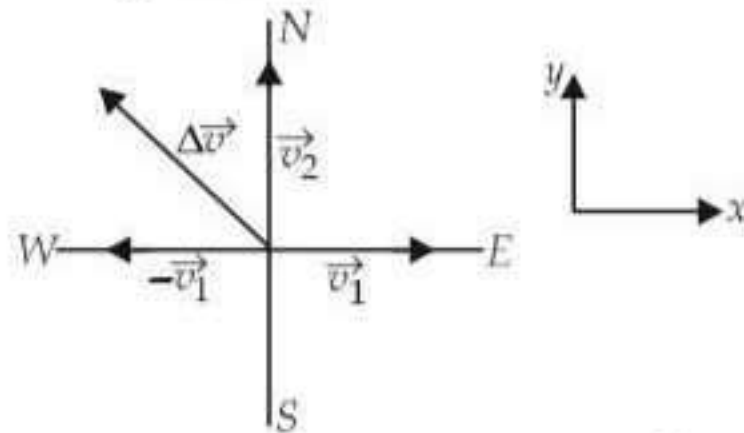
It acts towards the centre of the circle.

15. (a) : Here, $u = 20 \text{ m/s}$, $g = 10 \text{ m/s}^2$

For maximum range, angle of projection is $\theta = 45^\circ$

$$\begin{aligned} \therefore R_{\max} &= \frac{u^2 \sin 90^\circ}{g} = \frac{u^2}{g} \quad \left(\because R = \frac{u^2 \sin 2\theta}{g} \right) \\ &= \frac{(20 \text{ m/s})^2}{(10 \text{ m/s}^2)} = 40 \text{ m} \end{aligned}$$

16. (d) :



Velocity towards east direction, $\vec{v}_1 = 30 \hat{i} \text{ m/s}$

Velocity towards north direction, $\vec{v}_2 = 40 \hat{j} \text{ m/s}$

Change in velocity, $\Delta \vec{v} = \vec{v}_2 - \vec{v}_1 = (40 \hat{j} - 30 \hat{i})$

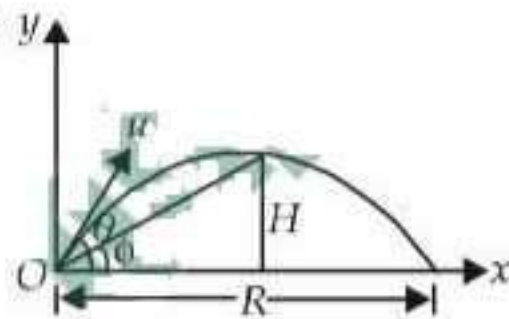
$\therefore |\Delta \vec{v}| = |40 \hat{j} - 30 \hat{i}| = 50 \text{ m/s}$

Average acceleration, $\vec{a}_{\text{av}} = \frac{\text{Change in velocity}}{\text{Time interval}}$

$$\vec{a}_{\text{av}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$$

$$|\vec{a}_{\text{av}}| = \frac{|\Delta \vec{v}|}{\Delta t} = \frac{50 \text{ m/s}}{10 \text{ s}} = 5 \text{ m/s}^2$$

17. (c) :



Let ϕ be elevation angle of the projectile at its highest point as seen from the point of projection O and θ be angle of projection with the horizontal.

$$\text{From figure, } \tan \phi = \frac{H}{R/2} \quad \dots(i)$$

In case of projectile motion

$$\text{Maximum height, } H = \frac{u^2 \sin^2 \theta}{2g}$$

$$\text{Horizontal range, } R = \frac{u^2 \sin 2\theta}{g}$$

Substituting these values of H and R in (i), we get

$$\tan \phi = \frac{\frac{u^2 \sin^2 \theta}{2g}}{\frac{u^2 \sin 2\theta}{g}}$$

$$\tan \phi = \frac{\sin^2 \theta}{\sin 2\theta} = \frac{\sin^2 \theta}{2 \sin \theta \cos \theta} = \frac{1}{2} \tan \theta$$

$$\tan \phi = \frac{1}{2} \tan 45^\circ = \frac{1}{2}$$

Here, $\theta = 45^\circ$

$$\therefore \tan \phi = \frac{1}{2} \tan 45^\circ = \frac{1}{2} \quad (\because \tan 45^\circ = 1)$$

$$\phi = \tan^{-1} \left(\frac{1}{2} \right)$$

18. (b) : Here,

Initial velocity, $\vec{u} = 3 \hat{i} + 4 \hat{j}$

Acceleration, $\vec{a} = 0.4 \hat{i} + 0.3 \hat{j}$

Time, $t = 10 \text{ s}$

Let \vec{v} be velocity of a particle after 10 s.

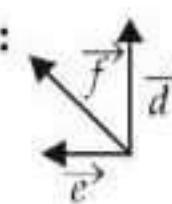
Using, $\vec{v} = \vec{u} + \vec{a}t$

$$\begin{aligned} \therefore \vec{v} &= (3 \hat{i} + 4 \hat{j}) + (0.4 \hat{i} + 0.3 \hat{j})(10) \\ &= 3 \hat{i} + 4 \hat{j} + 4 \hat{i} + 3 \hat{j} = 7 \hat{i} + 7 \hat{j} \end{aligned}$$

Speed of the particle after 10 s = $|\vec{v}|$

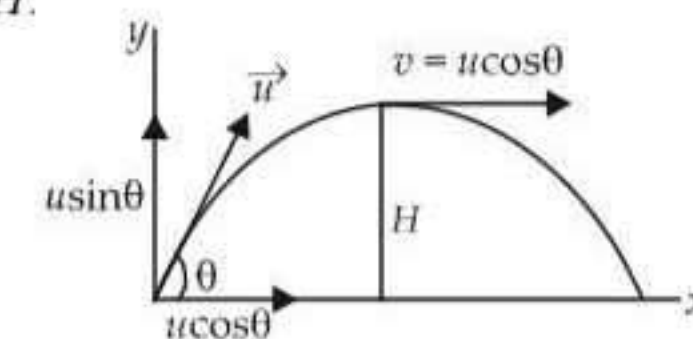
$$= \sqrt{(7)^2 + (7)^2} = 7\sqrt{2} \text{ units}$$

19. (c) :



From figure, $\vec{d} + \vec{e} = \vec{f}$

20. (a) : Let v be velocity of a projectile at maximum height H .



$$v = u \cos \theta$$

According to given problem, $v = \frac{u}{2}$

$$\therefore \frac{u}{2} = u \cos \theta \Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

$$\mathbf{21. (b) : } x = a \sin \omega t \quad \text{or} \quad \frac{x}{a} = \sin \omega t \quad \dots(i)$$

$$y = a \cos \omega t \quad \text{or} \quad \frac{y}{a} = \cos \omega t \quad \dots(ii)$$

Squaring and adding, we get

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1 \quad (\because \cos^2 \omega t + \sin^2 \omega t = 1)$$

or $x^2 + y^2 = a^2$

This is the equation of a circle. Hence particle follows a circular path.

22. (d) : Because the slope is highest at C,

$$v = \frac{ds}{dt} \text{ is maximum.}$$

23. (a) :



The horizontal momentum does not change. The change in vertical momentum is

$$mv \sin \theta - (-mv \sin \theta) = 2mv \frac{1}{\sqrt{2}} = \sqrt{2}mv$$

24. (d) : $|\vec{A} \times \vec{B}| = \sqrt{3}(\vec{A} \cdot \vec{B})$

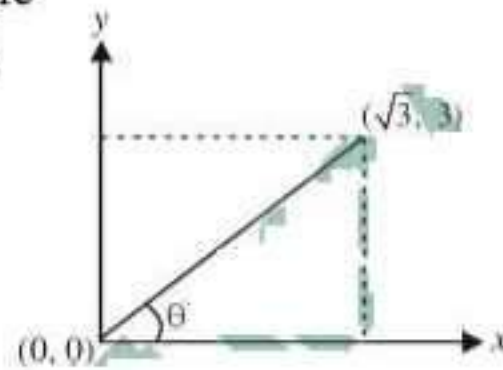
$$\therefore AB \sin \theta = \sqrt{3}AB \cos \theta$$

or, $\tan \theta = \sqrt{3}$ or, $\theta = \tan^{-1} \sqrt{3} = 60^\circ$.

25. (b) : Let θ be the angle which the particle makes with an x-axis.

From figure,

$$\tan \theta = \frac{3}{\sqrt{3}} = \sqrt{3}$$



or, $\theta = \tan^{-1}(\sqrt{3}) = 60^\circ$

26. (b) : Horizontal range $R = \frac{u^2 \sin 2\theta}{g}$

For angle of projection $(45^\circ - \theta)$, the horizontal range is

$$\therefore R_1 = \frac{u^2 \sin[2(45^\circ - \theta)]}{g} = \frac{u^2 \sin(90^\circ - 2\theta)}{g} = \frac{u^2 \cos 2\theta}{g}$$

For angle of projection $(45^\circ + \theta)$, the horizontal range is

$$R_2 = \frac{u^2 \sin[2(45^\circ + \theta)]}{g} = \frac{u^2 \sin(90^\circ + 2\theta)}{g} = \frac{u^2 \cos 2\theta}{g}$$

$$\therefore \frac{R_1}{R_2} = \frac{u^2 \cos 2\theta / g}{u^2 \cos 2\theta / g} = 1$$

\therefore The range is the same.

27. (b) : Let θ be angle between \vec{A} and \vec{B}

$$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|, \text{ then } |\vec{A} + \vec{B}|^2 = |\vec{A} - \vec{B}|^2$$

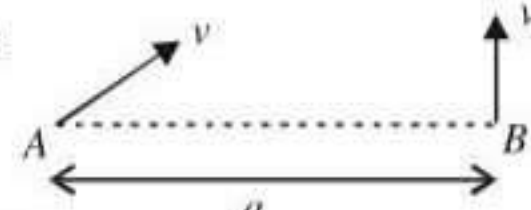
or $(\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B}) = (\vec{A} - \vec{B}) \cdot (\vec{A} - \vec{B})$

or $\vec{A} \cdot \vec{A} + \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{A} + \vec{B} \cdot \vec{B}$

$$= \vec{A} \cdot \vec{A} - \vec{A} \cdot \vec{B} - \vec{B} \cdot \vec{A} + \vec{B} \cdot \vec{B}$$

or $4AB \cos \theta = 0$ or $\cos \theta = 0^\circ$ or $\theta = 90^\circ$

28. (d) :



$$t = \frac{a}{v'} = \frac{a}{\sqrt{v^2 - v_1^2}}$$

29. (a, b) : $a = r\omega^2$; $\omega = 2\pi\nu$

22 revolution = 44 sec.

1 revolution = $44/22 = 2$ sec.

$\nu = 1/2$ Hz

$$a = r\omega^2 = 1 \times \frac{4\pi^2}{4} = \pi^2 \text{ m/s}^2.$$

Towards the centre, the centripetal acceleration $= -\omega^2 R$

and away from the centre, the centrifugal acceleration is $+\omega^2 R$.

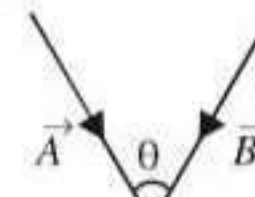
(a) and (b) are correct as the directions are given.

30. (d) : Let $\vec{A} \times \vec{B} = \vec{C}$

The cross product of \vec{B} and \vec{A} is perpendicular to the plane containing \vec{A} and \vec{B} i.e. perpendicular to \vec{A} . If a dot product of this cross

product and \vec{A} is taken, as the cross product is perpendicular to \vec{A} , $\vec{C} \cdot \vec{A} = 0$.

Therefore product of $(\vec{B} \times \vec{A}) \cdot \vec{A} = 0$.



31. (b) : $\vec{a} = 2\hat{i} + 3\hat{j} + 8\hat{k}$, $\vec{b} = 4\hat{j} - 4\hat{i} + \alpha\hat{k}$

$$\vec{a} \cdot \vec{b} = 0 \text{ if } \vec{a} \perp \vec{b}$$

$$(2\hat{i} + 3\hat{j} + 8\hat{k}) \cdot (-4\hat{i} + 4\hat{j} + \alpha\hat{k}) = 0$$

or, $-8 + 12 + 8\alpha = 0 \Rightarrow 4 + 8\alpha = 0$

$\Rightarrow \alpha = -1/2$.

32. (a) : $|\vec{A} \times \vec{B}| = \sqrt{3} \vec{A} \cdot \vec{B}$

$$|\vec{A}| |\vec{B}| \sin \theta = \sqrt{3} |\vec{A}| |\vec{B}| \cos \theta$$

$$\tan \theta = \sqrt{3} \Rightarrow \theta = 60^\circ$$

$$|\vec{A} + \vec{B}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}| |\vec{B}| \cos \theta} = (A^2 + B^2 + AB)^{1/2}$$

33. (b) : Given : $(\vec{F}_1 + \vec{F}_2) \perp (\vec{F}_1 - \vec{F}_2)$

$$\therefore (\vec{F}_1 + \vec{F}_2) \cdot (\vec{F}_1 - \vec{F}_2) = 0$$

$$F_1^2 - F_2^2 - \vec{F}_1 \cdot \vec{F}_2 + \vec{F}_2 \cdot \vec{F}_1 = 0 \Rightarrow F_1^2 = F_2^2$$

i.e. F_1, F_2 are equal to each other in magnitude.

34. (a) : Given :

$$r = \frac{20}{\pi} \text{ m}, v = 80 \text{ m/s}, \theta = 2 \text{ rev} = 4\pi \text{ rad.}$$

From equation $\omega^2 = \omega_0^2 + 2\alpha\theta$ ($\omega_0 = 0$)

$$\omega^2 = 2\alpha\theta \left(\omega = \frac{v}{r} \text{ and } a = r\alpha \right)$$

$$a = \frac{v^2}{2r\theta} = 40 \text{ m/s}^2.$$

35. (c) : Time required to reach the ground is dependent on the vertical motion of the particle. Vertical motion of both the particles A and B are exactly same. Although particle B has an initial velocity, but that is in horizontal direction and it has no component in vertical (component of a vector at a direction of $90^\circ = 0$) direction. Hence they will reach the ground simultaneously.

36. (b) : Mass, $m = 3 \text{ kg}$, force, $F = 6t^2\hat{i} + 4t\hat{j}$
 \therefore acceleration,

$$a = F/m = \frac{6t^2\hat{i} + 4t\hat{j}}{3} = 2t^2\hat{i} + \frac{4}{3}t\hat{j}$$

Now, $a = \frac{dv}{dt} = 2t^2\hat{i} + \frac{4}{3}t\hat{j}$;

$$\therefore dv = \left(2t^2\hat{i} + \frac{4}{3}t\hat{j} \right) dt \quad \therefore v = \int_0^3 \left(2t^2\hat{i} + \frac{4}{3}t\hat{j} \right) dt$$

$$= \frac{2}{3}t^3\hat{i} + \frac{4}{6}t^2\hat{j} \Big|_0^3 = 18\hat{i} + 6\hat{j}.$$

37. (c) : $|\vec{A} + \vec{B}| = |\vec{A}| + |\vec{B}|$ if $\vec{A} \parallel \vec{B}$, $\theta = 0^\circ$.

38. (c) : $\omega = \frac{2\pi}{t}$, t is same $\therefore \frac{\omega_1}{\omega_2} = 1$

39. (a) : $v_{\text{Resultant}} = \frac{1 \text{ km}}{1/4 \text{ hr}} = 4 \text{ km/hr}$

$$\therefore v_{\text{River}} = \sqrt{5^2 - 4^2} = 3 \text{ km/hr}$$

40. (b) : As $\theta_2 = 90^\circ = \theta_1$,

So range of projectile,

$$R_1 = \frac{v_0^2 \sin 2\theta}{g} = \frac{v_0^2 2 \sin \theta \cos \theta}{g}$$

$$R_2 = \frac{v_0^2 2 \sin(90 - \theta_1) \cos(90 - \theta_1)}{g}$$

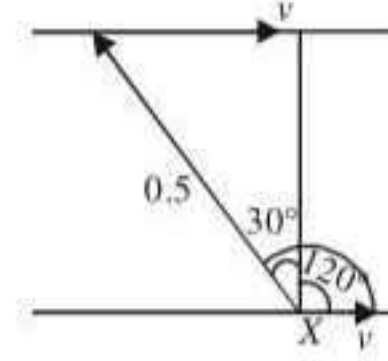
$$R_2 = \frac{v_0^2 2 \cos \theta_1 \sin \theta_1}{g} = R_1$$

41. (c) : Vertical acceleration in both the cases is g , whereas horizontal velocity is constant.

42. (a) : $F_{\text{centripetal}} = \frac{mv^2}{R}$; $v = \left(36 \times \frac{5}{18} \right) \text{ m/s}$

$$F_{\text{centripetal}} = \frac{500 \times \left(36 \times \frac{5}{18} \right)^2}{50} = 1000 \text{ N}$$

43. (a) : Let v be the velocity of river water. As shown in figure,



$$\sin 30^\circ = \frac{v}{0.5}$$

or, $v = 0.5 \sin 30^\circ$

$$= 0.5 \times (1/2) = 0.25 \text{ m/s.}$$

44. (c) : $t = \frac{2\pi}{\omega_1} = \frac{2\pi}{\omega_2} \Rightarrow \frac{\omega_1}{\omega_2} = 1$

45. (b) : For a unit vector \hat{n} , $|\hat{n}| = 1$

$$|0.5\hat{i} - 0.8\hat{j} + c\hat{k}|^2 = 1^2 \Rightarrow 0.25 + 0.64 + c^2 = 1$$

or $c = \sqrt{0.11}$

46. (b) : $\vec{v} = \vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -6 & 6 \\ 3 & -4 & 1 \end{vmatrix} = 18\hat{i} + 13\hat{j} - 2\hat{k}$

47. (d) : Let particle B move upwards with velocity

$$v, \text{ then } \tan 60^\circ = \frac{v}{10}; \quad v = \sqrt{3} \times 10 = 17.3 \text{ m/s.}$$

48. (c) : $\frac{mv^2}{r} = 25$; $v = \sqrt{\frac{25 \times 1.96}{0.25}} = 14 \text{ m/s.}$

49. (b) : Since the angular momentum has both magnitude and direction, it is a vector quantity.

50. (d) : Radius of circle (r) = 20 cm = 0.2 m and angular velocity (ω) = 10 rad/s.

linear velocity (v) = $r\omega = 0.2 \times 10 = 2 \text{ m/s.}$

51. (d) : Position vector of the particle

$$\vec{r} = (a \cos \omega t)\hat{i} + (a \sin \omega t)\hat{j}$$

velocity vector

$$\vec{v} = \frac{d\vec{r}}{dt} = (-a\omega \sin \omega t)\hat{i} + (a\omega \cos \omega t)\hat{j}$$

$$= \omega [(-a \sin \omega t)\hat{i} + (a \cos \omega t)\hat{j}]$$

$$\vec{v} \cdot \vec{r} = \omega [(-a \sin \omega t)\hat{i} + (a \cos \omega t)\hat{j}] \cdot [(a \cos \omega t)\hat{i} + (a \sin \omega t)\hat{j}]$$

$$= \omega [-a^2 \sin \omega t \cos \omega t + a^2 \cos \omega t \sin \omega t] = 0$$

Therefore velocity vector is perpendicular to the displacement vector.

52. (a) : Number of revolutions per minute (n) = 120.

Therefore angular speed (ω)

$$= \frac{2\pi n}{60} = \frac{2\pi \times 120}{60} = 4\pi \text{ rad/s.}$$

53. (a) : $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{B} = 3\hat{i} + 4\hat{j} - 5\hat{k}$.

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

$$= \frac{(3\hat{i} + 4\hat{j} + 5\hat{k}) \cdot (3\hat{i} + 4\hat{j} - 5\hat{k})}{[\sqrt{(3)^2 + (4)^2 + (5)^2}] \times [\sqrt{(3)^2 + (4)^2 + (5)^2}]}$$

$$= \frac{9 + 16 - 25}{50} = 0 \text{ or } \theta = 90^\circ.$$

54. (b) : Let the velocity of river be v_R and velocity of boat is v_B

$$\therefore \text{Resultant velocity} = \sqrt{v_B^2 + v_R^2 + 2v_B v_R \cos \theta}$$

$$(10) = \sqrt{v_B^2 + v_R^2 + 2v_B v_R \cos 90^\circ}$$

$$(10) = \sqrt{(8)^2 + v_R^2} \text{ or } (10)^2 = (8)^2 + v_R^2$$

$$v_R^2 = 100 - 64 \text{ or } v_R = 6 \text{ km/hr}$$

55. (b) : For the given velocity of projection u , the horizontal range is the same for the angle of projection θ and $90^\circ - \theta$

$$\text{Horizontal range } R = \frac{u^2 \sin 2\theta}{g}$$

$$\therefore \text{For body A, } R_A = \frac{u^2 \sin(2 \times 30^\circ)}{g} = \frac{u^2 \sin 60^\circ}{g}$$

$$\text{For body B, } R_B = \frac{u^2 \sin(2 \times 60^\circ)}{g}$$

$$R_B = \frac{u^2 \sin 120^\circ}{g} = \frac{u^2 \sin(180^\circ - 60^\circ)}{g} = \frac{u^2 \sin 60^\circ}{g}$$

The range is the same whether the angle is θ or $90^\circ - \theta$.

\therefore The ratio of ranges is 1 : 1

56. (c) : The cross product $\vec{A} \times \vec{B}$ is a vector, with its direction perpendicular to both \vec{A} and \vec{B} . $\vec{A} \times \vec{B}$ is area. If side B is zero, area is zero.

$\vec{A} \times 0$ is a zero vector.

If in case 0 is a scalar, then also the product is zero. But a scalar \times a vector is also a vector.

Hence one gets a zero vector in any case.

57. (b) : Frequency of rotation $\nu = 120 \text{ rpm} = 2 \text{ rps}$
length of blade $r = 30 \text{ cm} = 0.3 \text{ m}$
Centripetal acceleration $a = \omega^2 r = (2\pi\nu)^2 r$

$$= 4\pi^2 \nu^2 r = 4\pi^2 (2)^2 (0.3) = 47.4 \text{ ms}^{-2}$$

58. (c) : Horizontal range $R = \frac{u^2 \sin 2\theta}{g}$

For maximum horizontal range $\theta = 45^\circ$

or $R_m = \frac{u^2}{g}$

where u be muzzle velocity of a shell

$$\therefore (1600 \text{ m}) = \frac{u^2}{(10 \text{ ms}^{-2})^2} \text{ or } u = 400 \text{ m s}^{-1}$$

59. (a) : $v_1 = 50 \text{ km/hr}$ due north

$v_2 = 50 \text{ km/hr}$ due west

$-v_1 = 50 \text{ km/hr}$ due south

Magnitude of change in velocity

$$= |\vec{v}_2 - \vec{v}_1| = |\vec{v}_2 + (-\vec{v}_1)|$$

$$= \sqrt{v_2^2 + (-v_1)^2}$$

$$= \sqrt{(50)^2 + (50)^2} = 70.7 \text{ km/hr}$$

$\vec{v} = 70.7 \text{ km/hr}$ along south-west direction

60. (a) : Let θ be angle between \vec{A} and \vec{B}

Given : $A = |\vec{A}| = 3 \text{ units}$

$B = |\vec{B}| = 4 \text{ units}$

$C = |\vec{C}| = 5 \text{ units}$

$$\vec{A} + \vec{B} = \vec{C}$$

$$\therefore (\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B}) = \vec{C} \cdot \vec{C}$$

$$\vec{A} \cdot \vec{A} + \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{A} + \vec{B} \cdot \vec{B} = \vec{C} \cdot \vec{C}$$

$$A^2 + 2AB\cos\theta + B^2 = C^2$$

$$9 + 2AB\cos\theta + 16 = 25 \text{ or } 2AB\cos\theta = 0$$

or $\cos\theta = 0 \therefore \theta = 90^\circ$.

61. (d) : Choose the positive direction of x-axis to be from south to north. Then

Velocity of train $v_T = +10 \text{ m s}^{-1}$

Velocity of parrot $v_P = -5 \text{ m s}^{-1}$

Relative velocity of parrot with respect to train

$$= v_P - v_T = (-5 \text{ ms}^{-1}) - (+10 \text{ ms}^{-1}) = -15 \text{ m s}^{-1}$$

i.e. parrot appears to move with a speed of 15 m s^{-1} from north to south

\therefore Time taken by parrot to cross the train

$$= \frac{150 \text{ m}}{15 \text{ m s}^{-1}} = 10 \text{ s}$$

